G485 Fields, Particles and the Frontiers of Physics

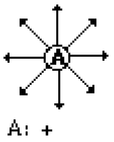
Electric and Magnetic Fields

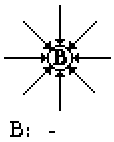
Electric Fields

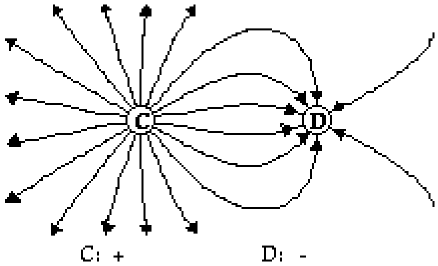
Electric fields are created by **electric charges**. If a charged particle ventures into this electric field, then it will experience a force. The magnitude of which depends of the charge and strength of the electric field. **Electric Field Strength, E, at a point is the force experienced per unit charge exerted on a positive charge placed at that point.**

$$\Rightarrow \boldsymbol{E} = \frac{\boldsymbol{F}}{\boldsymbol{Q}}$$

Where F is the force experienced by a positive charge of magnitude Q.







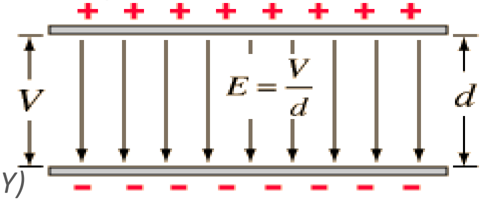
Electric field patterns can be mapped out using **electric field lines**. The direction of the field at a point in space shows the direction of the force experienced by a small positive charge. Hence electric fields point away from positively charged objects and towards negatively charged objects.

A **uniform electric field** is created when two oppositely charged parallel plates mean the **electric field strength is constant** and the electric field pattern has parallel lines which are evenly spaced. The p.d. between the plates is V and they are separated by distance d. A positive charge Q moves across. E is electric field strength.

$$\text{Work done on charge} = \text{energy transformed} \quad F \times d = V \times Q$$

$$\Rightarrow \frac{F}{Q} = \frac{V}{d} \text{ but } \frac{F}{Q} = E \text{ so} \Rightarrow \boldsymbol{E} = \frac{\boldsymbol{V}}{\boldsymbol{d}}$$

*(for **parallel** plates ONLY)*



Example

An electron is accelerated between 2 charged parallel plates. The p.d. is 450v between them and 1.8cm apart. The electron goes from negative to positive. What is the force it experiences due to the electric field, its acceleration and its final velocity?

$E = V/d$ so $E = 450 / (1.8 \times 10^{-2}) = 2.5 \times 10^{4}\ \text{Vm}^{-1}$. $EQ = F \Rightarrow 2.5 \times 10^{4} \times 1.6 \times 10^{-19} = 4 \times 10^{-15}$ N. $F/m = a \Rightarrow 4 \times 10^{-15}/(9.11 \times 10^{-31}) = 4.39 \times 10^{15}\ \text{ms}^{-1}$ $v^{2} = 2 \times 4.39 \times 10^{15} \times 1.8 \times 10^{-2} \Rightarrow v = 1.3 \times 10^{7}\ \text{ms}^{-1}$.

Coulomb's Law states **two point charges exert an electrical force on each other that is directly proportional to the product of the charges and inversely proportional to the square of separation between them.**

i.e. $\boldsymbol{F} \propto \boldsymbol{Qq}$ and $\boldsymbol{F} \propto \frac{1}{r^2}$

$$\boldsymbol{F} = \frac{\boldsymbol{Qq}}{\boldsymbol{4\pi\varepsilon_0 r^2}}$$

$\boldsymbol{\varepsilon_0}$ is the permittivity of free space. It has an experimental value of $8.85 \times 10^{-12}\ \text{F m}^{-1}$.

Using $\boldsymbol{E} = \frac{\boldsymbol{F}}{\boldsymbol{Q}}$;

$$\Rightarrow \boldsymbol{E} = \frac{\boldsymbol{Q}}{\boldsymbol{4\pi\varepsilon_0 r^2}} \text{; hence } \boldsymbol{E} \propto \frac{1}{r^2}$$

Comparing **electric** and **gravitational** fields, the similarities are:

- Both electric and gravitational fields are to do with 'action at a distance'
- Field strengths follow an inverse square law with distance.
- A point mass and point charge both produce radial fields
- Field strength is defined as force per unit mass or positive charge.

The differences are

- Electric field is created by charge, whilst a gravitational field by mass.
- Electric fields can be attractive or repulsive. Gravitational fields are always attractive.

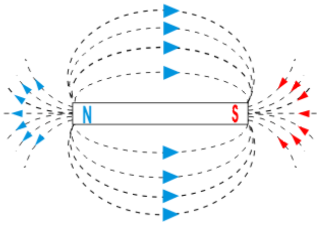
Example

Two identical spheres are charged. Each sphere has a radius 2.5cm with charge +0.3nC. Find the force between the charged spheres when their surfaces are separated by a distance of 2cm.

F= Qq/4pier^2 = (0.3x10^{-9})^2/4pix8.85x10^{-12}x(7x10^{-2})^2 = 1.65x10^{-7}N

Magnetic Fields

Magnetic fields will produce a force on another magnet in this field. Magnetic fields are caused by tiny magnetic fields created by electrons moving around in the atoms of the material of the magnet. Field lines go from north to south.



Fleming's Left Hand Rule

Closely placed **magnets** experience a force because their **magnetic fields interact**. A **current**-carrying wire experiences a force when placed in an external magnetic field of a magnet because their **magnetic fields** also **interact**. The direction of the force experienced by a current carrying conductor placed at right angles to a magnetic field can be determined using **Fleming's left-hand rule**.

The strength of the magnetic field is stronger where the magnetic fields lines are close together. The strength of the magnetic field is **magnetic flux density**, ***B***. i.e. the **force experienced by a current-carrying wire placed at right angles to a magnetic field.**

$\boldsymbol{B} = \frac{F}{IL}$ where F is the force on the current-carrying wire, I the current in the wire and L the length of the wire in the magnetic field. Unit of Tesla; T.

The magnetic flux density is 1 T when a wire carrying a current of 1 A placed at right angles to the field experiences a force of 1 N per metre of its length.

Therefore the force on a current-carrying wire placed at 90^o to an external magnetic field is given by $F = BIL$ and by extension, $F = BIL \sin \emptyset$ where $\emptyset$ is the angle between wire and field.

Example

A thin wire has a weight of 1.6x10^{-3}Ncm^{-1}. The wire is placed at right angles to the magnetic field of flux density 0.2 T. The direction of the current in the wire is such that it experiences an upward force. The current is slowly increased. Calculate the current in the wire when the force on the wire due to the magnetic field is equal to the weight of the wire.

For a 1cm length of wire, the force is 1.6x10^{-3}. 6x10^{-3}= 0.2 x I x 10^{-2} => I needs to be 0.8A.

Moving charges in a magnetic field

A **current**-**carrying wire** placed in a **magnetic field** experiences a **force** because each moving electron within the wire experiences a tiny force.

Consider the force F on a **positive particle** moving is at **right** angles to the **magnetic field**. $F = BQv$ where B is the **magnetic flux density**, Q is the charge on the particle and v the speed of the particle. A higher value of B has field lines closer together. The direction of the force is determined by Flemings Left Hand rule.

Hence a stronger field, greater charge and faster velocity give a bigger force. *For the more specific case of an electron, where* $Q = e; F = Bev$

Circular motion can be observed when an electron enters a region of uniform magnetic field. The force on the electron is at right-angles to its velocity. Combining F=BQv and F=mv^2/r ⟹ $r = \frac{mv}{BQ}$

Mass Spectrometer

Used to determine the **masses** of **charged ions** and their **relative abundances**.

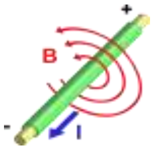
- A **uniform magnetic field** is used to **deflect** the **charged ions** in a **circular path** in an **evacuated chamber**.
- A moveable **detector** used to find the **radius** of the path and the **relative abundance** of the ions.
- The radius r of the path in the magnetic field region is $r = \frac{mv}{BQ}$.
- The radius that the particle moves in (and hence the place where it hits the sensor) depends on the different **mass to charge ratio**.

Example

An electron travelling at 7.5x10^6 ms^{-1} enters a region of uniform magnetic field of flux density 60microT. The electron is initially travelling at right angles to the magnetic field. Find the radius of the circular motion

The magnetic force provides the centripetal acceleration. Bev = mv^2*/r => 7.5x10*^{6} *x 1.6x10*$^{-19}$ *x 7.5x10*^{6} *= 9.11x10*$^{-31}$ *x (7.5x10*^{6}*)*^{2} */ r => r= 0.71m*

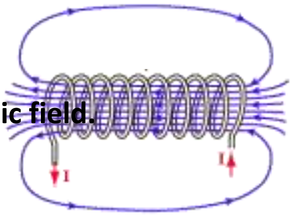
Electromagnetism



Moving charges create a magnetic field. The magnetic field disappears when the charges stop moving. Hence, a magnetic field can be created by an **electric current** in a wire. The right hand grip rule dictates the direction of the force.

Magnetic flux ϕ is defined by

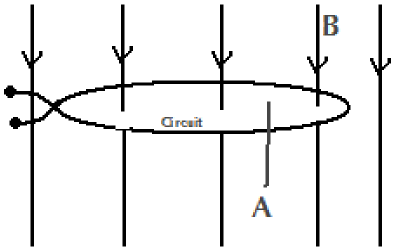
magnetic flux = **magnetic flux density x cross-sectional area normal to magnetic field.**



The magnetic field is concentrated into a nearly uniform field in the center of a long solenoid. The field outside is weak and divergent.

$$\boldsymbol{\phi = BA\cos\theta}$$

The unit for magnetic flux is the weber. **A magnetic field of flux density 1 T passing normally through an area of 1m^2 produces a magnetic flux equal to 1 Wb.**



Magnetic flux linkage = number of turns x magnetic flux

$$\boldsymbol{Magnetic\ Flux\ Linkage = N\phi = NBA\cos\theta}$$

And hence the unit is also the weber.

- An e.m.f is **induced** in a circuit whenever there is a **change** in the **magnetic flux linkage**. The magnitude of the induced e.m.f in a circuit can be determined using **Faraday's law of electromagnetic induction;**

The magnitude of the induced e.m.f is equal to the rate of change of Magnetic flux linkage.

i.e.;

$$E = -\frac{\Delta(N\phi)}{\Delta t}$$

There are 3 ways in which an e.m.f may be induced in a circuit:

- Change the magnetic flux density B (e.g. move a coil closer to the pole of a bar magnet)
- Change the area A of the circuit (e.g. move a straight wire at right angles to the magnetic field)
- Change the angle (e.g. rotate the coil)

The direction of the induced e.m.f in a circuit is governed by **Lenz's law;**

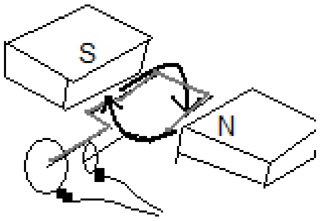
The direction of induced e.m.f. or the current is such to oppose the change that is producing it. This is the reason of the negative sign of the above equation. Fleming's right-hand rule is applicable here.

Example

A flat coil of 800 turns has cross-sectional area $7x10^{-4}m^2$ and is connected to the terminals of an ammeter. The total resistance of the coil and the ammeter is 0.3 Ω. The plane of the coil is at right angles to a magnetic field of flux density 0.12T. The coil is removed from the magnetic field in 50 ms. Find the average induced e.m.f. across the coil an the average current shown by the ammeter.

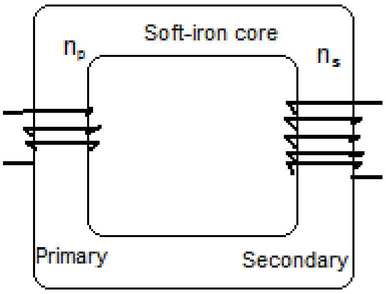
$E = -\frac{\Delta(N\Phi)}{\Delta t}$ => E.m.f = $(-0.12x800x7x10^{-4})/(50x10^{-2})$ = 1.34 V Av current = e.m.f/resistance = 1.34/0.3 = 4.5A

A simple **generator**



- They convert **kinetic** energy to **electrical** energy. I.e. they **induce** an **electric current** by **rotating** a **coil** in a **magnetic field**.
- A coil **rotates** at a constant frequency in a **magnetic field**. The rotation of the coil **induces** an **e.m.f.** across the ends of the coil; some of the coil's **kinetic energy** is transformed into **electrical energy**. The e.m.f induced across the ends of the coil is equal the **rate** of **change** of **flux linkage**. The output e.m.f. and current changes direction with every half rotation of the coil producing an **alternating current (AC).**

A **transformer**



- **They change the size** of voltage for an **alternating current.**
- Consists of two coils mounted on an iron core. The **alternating current** in the **input** coil produces **magnetic flux** in the iron core.
- The **magnetic field** is passed through the iron core to the output coil where it **induces** an **alternating voltage** of the same frequency.

$\therefore \frac{V_s}{V_p} = \frac{n_s}{n_p}$ where v is input and output **voltage** and n is **number of turns** on primary and secondary wire.

For a step down transformer $n_s > n_p$ and step-up; $n_s < n_p$.

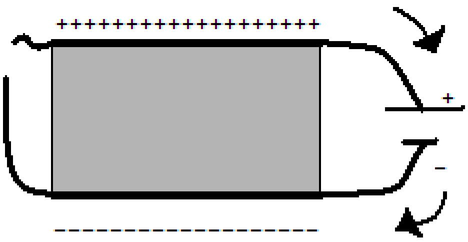
For a 100% efficient transformer; $input\ power = output\ power \qquad \Longrightarrow V_p I_p = V_s I_s$

P=IV means low current => high voltage. Therefore step up transformers are used for the national grid.

Capacitors

Properties

A capacitor consists of two metal plates separated by insulation. When connected to a source of e.m.f., the plates acquire *equal* but *opposite* charges. The positive plate loses electrons and the negative plate gains an equal number of electrons.



The **magnitude** of the **charge** Q on **one** of the **plates** is **directly proportional** to the **p.d**. V across the capacitor. i.e.

$Q \propto V$ The constant relies on the capacitor

i.e. $Q = VC$ where C is the **capacitance**.

$\Longrightarrow$ Hence **capacitance is charge stored per volt.**

And hence the unit of a farad can be defined. **A capacitor has a capacitance of 1 F when it can store 1 coulomb of charge per unit volt.**

Example
The p.d. across a 1000μF capacitor is changed from 2V to 6V. Find the charge in mC gained by each capacitor plate and the number of electrons gained by the negative plate.
Q = VC; Initial charge = 1000x10^{-6}*x2= 2000*μ*C ; Final Q = 1000x*10^{-6}*x6 = 6000*μ*C. ΔQ= 4000*μ*C 4000x*10^{-6}*/(1.6x*10^{-19}*) = 2.5x*10^{16}*.*

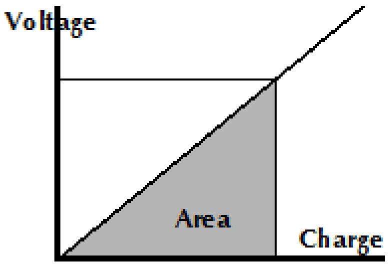
In **series**, the **charge** on each capacitor is the **same**. The total capacitance C is given by

$$C_{total} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

The total p.d. is given by $V_{total} = V_1 + V_2 + V_3 + \cdots$

In **parallel,** the total **charge** stored by the arrangement of capacitors is the sum of the charges in each individual capacitor i.e. $Q_{total} = Q_1 + Q_2 + Q_3 + \cdots$
The total **capacitance** is given by $C_{total} = C_1 + C_2 + C_3 + \cdots$
The **voltage** across each capacitor is the same.

Energy stored by a capacitor



Work has to be done **against** the repulsive forces to increase the number of electrons on the negative capacitor plate. Since work is done, this implies a capacitor will store electrical energy.

The Q=VC relationship implies that the **reciprocal** of the gradient is equal to the **capacitance** of this graph. The area is equivalent to **work done** due to W=VQ.

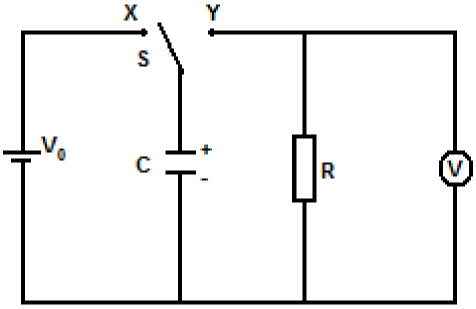
Since C is a constant, the work done, W satisfies $W = \frac{1}{2}QV$ and hence $W = \frac{1}{2}CV^2$

Example

A 0.02F capacitor is charged to 9V. The fully charged capacitor is discharged through a lamp in a time of 40 ms. Find the average power dissipated by the lamp.

E = 0.5CV^2 = 0.5x0.02x9 = 0.81J. Power= Work/Time => Power = 0.81/(40x10^{-2}) = 20W

Capacitor discharging through a resistor



With the switch S at position X the capacitor is fully charged. The p.d. across it is V_0. When the switch is moved to Y, the capacitor will discharge through the resistor.

The capacitor is parallel with the resistor, hence the p.d. across the capacitor and the resistor will always be the same. The charge Q on the capacitor will decrease exponentially with time.

Since Q = VC and V=IR the p.d. across the capacitor and the current in the circuit thus follow the same exponential decay.

$$\therefore \quad Q = Q_o e^{-\frac{t}{CR}}$$

$$\& \quad V = V_o e^{-\frac{t}{CR}}$$

$$\& \quad I = I_o e^{-\frac{t}{CR}}$$

The graphs of Q against t, V against t and I against t hence all look the same. For each individual graph, the area of gradient may relate to a quantity depending on equations. E.g. Q=It

With exponential decay, in a **given time interval**, the **quantity** is a **fraction** of the size regardless of when the time interval is. (constant-ratio property)

- When t= CR , the charge left on the capacitor is $e^{-1} \approx 0.37...$ of the initial charge.

  Time constant = CR The time constant for a capacitor-resistor circuit is defined as the time taken for the charge (or the current or p.d.) to decrease to 0.37... of its previous value.

Capacitors can be used in

- **Flash photography;** the charge flows into xenon gas which emits a bright light.
- **Nuclear fusion**; capacitors release a large charge to deliver large amount of energy via lasers.
- **Back-up power supplies;** computers use them to ensure no data is lost in a power cut.

Example

A capacitor of capacitance 120μF is charged to 10V. It is discharged through a resistor of resistance 100kΩ. Find the initial current and the current after 24s.

V=IR => 10= I x 100x10^3 => I= 10^{-4} A. I = $I_0 e^{-\frac{t}{CR}}$ => I = 100xe$^{(-24/12)}$= 13.5 μA

Nuclear Physics

The nuclear atom structure

- Rutherford carried out the electron scattering experiment. Alpha particles were targeted towards a thing gold foil. The number of alpha particles scattered at various angles were counted using a alpha particle detector.

  Most of the alpha particles weren't scattered ⇒ The gold atoms are **mostly empty space**

  Some of the alpha particles were scattered through large radius ⇒ **Small dense positive** nucleus.

  Using coulombs law ($F = \frac{Qq}{4\pi\varepsilon_0 r^2}$), we can work out the force at a given distance.

The nucleus of an atom is small and dense. Nuclei (of all elements) tend to have a density of **10^{17}kgms^{-3}**. This is far greater than ordinary matter. Nucleus is 1/10 000 the size of a whole atom.

- The positively charged protons inside the nucleus of an atom **repel** each other (**coulombs law** can work this out). The **attractive gravitational** force us far too weak. The attractive force is called the **strong nuclear force**. This force is an attractive force and is a very short-range force.

  *A **nucleon** is a proton or neutron and hence the nucleon number is the mass number. A nuclide is a particular combination of neutrons and protons.*

Fundamental Particles

Fundamental particles cannot be subdivided into smaller constituents so they have no internal structure. Quarks and electrons are examples.

- **Hadrons**: (groups of particles made of quarks) All hadrons are **affected** by the **strong nuclear force**. The range is about 10^{-14}m. Protons, neutrons and mesons are examples. Hence protons and neutrons are not fundamental particles.
- **Leptons:** are **not affected** by **strong nuclear force**. Electrons, neutrinos and muons are examples.

Quarks account for the properties (mass, charge, spin etc.) of hadrons. In the model, there are 6 quarks and 6 anti-quarks. The properties are described by charge, Q, baryon number B and strangeness S.

Quark	Q (in units of e)	B (Baryon number)	S (Strangeness)
Up (u)	$+\frac{2}{3}$	$+\frac{1}{3}$	0
Down (d)	$-\frac{1}{3}$	$+\frac{1}{3}$	0
Top (t)	$+\frac{2}{3}$	$+\frac{1}{3}$	0
Bottom (b)	$-\frac{1}{3}$	$+\frac{1}{3}$	0
Charm (c)	$+\frac{2}{3}$	$+\frac{1}{3}$	0
Strange (s)	$-\frac{1}{3}$	$+\frac{1}{3}$	-1

An anti-quark has a bar over the symbol e.g. $\bar{d}$

- A **proton** has two up quarks and 1 down quark; **(u u d)**
- A **neutron** has one up quark and two down quarks; **(u d d)**
- *A **pi$^+$ meson** has one up quark and one down antiquark;* ***(u $\overline{d}$)***

You can hence work out the charge, baryon number and strangeness of the proton, neutron and pi$^+$ meson. E.g. proton has Q=1, B=1, S=0; neutron has Q=0, B=1, S=0; pi$^+$ meson has Q=1, B=0, S=0.

Q,B,S are conserved in all nuclear reactions.

In beta-minus decay, a neutron is changed into a proton- **udd** becomes **uud**. Weak interaction changes this d quark into a u quark. In beta-plus, **uud** becomes **udd** so u becomes d.

Radioactivity

Natural radioactivity is the ***spontaneous*** *(unaffected by external conditions or presence of other nuclei)* and ***random*** *(unpredictable when a nucleus will decay)* disintegration of unstable nuclei because they have surplus energy. In the process of disintegration, a nucleus emits a particle (alpha particle, α, beta-minus, β^-, or beta-plus, β^+) and/or a gamma ray photon γ.

Alpha particle, α

- **Nucleus of helium** with **charge +2e (^4_2He)**
- Emitted with the **same** kinetic energy, with a speed of about 10^6ms^{-1}
- Are **very ionising**.
- Have a **short range of a few cm in air.**
- Deflected by electric and magnetic fields.

Smoke detectors use a alpha source to detect presence of smoke particles.

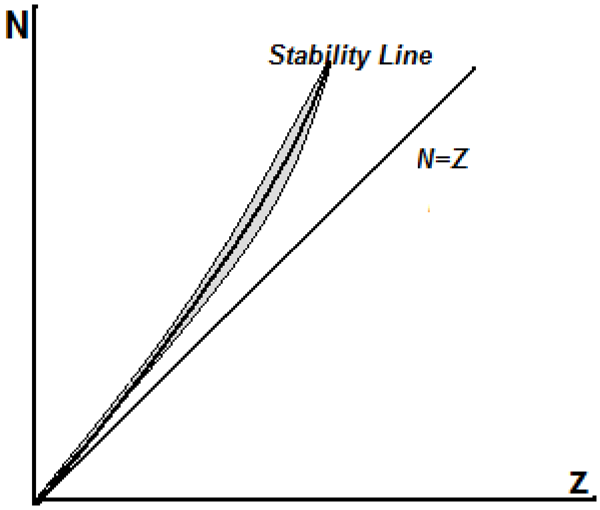
Beta-minus particle, β^-

- **Electron with charge –e (e$^-$)**
- Emitted with a **range** of kinetic energies, with a speed of about 10^7
- Are **fairly ionising.**
- Have a range of **a few mm of aluminium**
- Deflected by electric and magnetic fields.

Gamma Rays, γ

- **Short-ranged EM waves**
- Have **no charge**
- Travel at c ms^{-1}
- **Weakly ionising**
- Range of **a few cm in lead**
- Not affected by electric or magnetic fields

The **stability** of the nucleus is linked to the relative number of neutrons, N, and the number of protons, Z.



Stable nuclei lie on the line of stability. The nuclei in the shaded region are unstable. Decay of an unstable nucleus brings it close to the line of stability.

Alpha decay

A helium nucleus is emitted;

$^{232}_{90}Th \longrightarrow {}^{228}_{88}Ra + {}^{4}_{2}He$

Beta-minus decay

Electron-rich nuclei emit an electron

$^{90}_{38}Sr \longrightarrow {}^{90}_{39}Y + {}^{0}_{-1}e\ (+\bar{v})$

The weak nuclear force is responsible for this decay.

A **neutron** transforms into a **proton**, **electron** and **antineutrino** (very little charge and mass)

Beta-plus decay

Some proton-rich nuclei emit a positron.

$^{15}_{8}O \longrightarrow {}^{15}_{7}N + {}^{0}_{+1}e\ (+v)$

The weak nuclear force is responsible for this decay.

A **proton** transforms into a **neutron**, **positron** and **neutrino**.

The activity (A) of a sample is the rate at which the nuclei decay or disintegrate. (SI unit Becquerel Bq)

The decay constant (λ) is the probability of decay of a nucleus per unit time.

$$\boldsymbol{A = \lambda N}$$

The activity A of a sample decays exponentially with time. Hence does N and the count rate (C). Hence;

$$\boldsymbol{A = A_0 e^{-\lambda t}} \qquad \boldsymbol{N = N_0 e^{-\lambda t}} \qquad \boldsymbol{C = C_0 e^{-\lambda t}}$$

The **half-life** of an isotope is the average time taken for half of the active nuclei to decay.

Carbon-14 has a half life of 5570 years; and hence is used in carbon dating.

$\boldsymbol{N = N_0 e^{-\lambda t_{\frac{1}{2}}}}$ **when N=** $\boldsymbol{\frac{N_0}{2} \Longrightarrow \frac{N}{2} = N\, e^{-\lambda t_{\frac{1}{2}}} \Longrightarrow}$ $\boldsymbol{\lambda\, t_{\frac{1}{2}} = ln(2) \approx 0.693}$ **...**

Nuclear Binding energy

The mass-energy equation is

$$\Delta E = \Delta mc^2$$

where ΔE is the change in energy of the system, Δm is the change in mass of the system and c = 3x10^{8}ms^{-1}.
Consequence of this:

- The mass of a system *increases* when external energy is supplied to the system.
- Energy is *released* from the system when its mass *decreases*.

The 'system' could be decaying radioactive nuclei, an accelerated electron, a person etc. But the changes in mass a really small. The change in mass of a high-speed particle is much more significant. An electron travelling at 20% the speed of light will have a 2% increase in mass.

We can use this mass-energy equation to understand how a nucleus emits energy.
The total mass of the $^{14}_{6}C \longrightarrow {}^{14}_{7}N + {}^{0}_{-1}e + \bar{v}$ system is **slightly lower** on the product side. Hence a decrease in mass (the negativity of the mass change is irrelevant) implies a loss of energy as kinetic energy. In this case the increase in energy is kinetic, as the nitrogen-14 nucleus recoils whilst the electron and neutrino fly in the opposite direction (hence also satisfying conservation of momentum).

Nuclear binding energy

For all nuclei, it is found that: $mass\ of\ nucleus < total\ mass\ of\ protons\ and\ neutrons$
The difference in mass is known as **mass defect** and this is linked to the **binding energy** of the nucleus.

The binding energy of the nucleus is the minimum energy needed to separate all its nucleons.

In order to compare stability of the different nuclei, we consider the **binding energy per nucleon.**

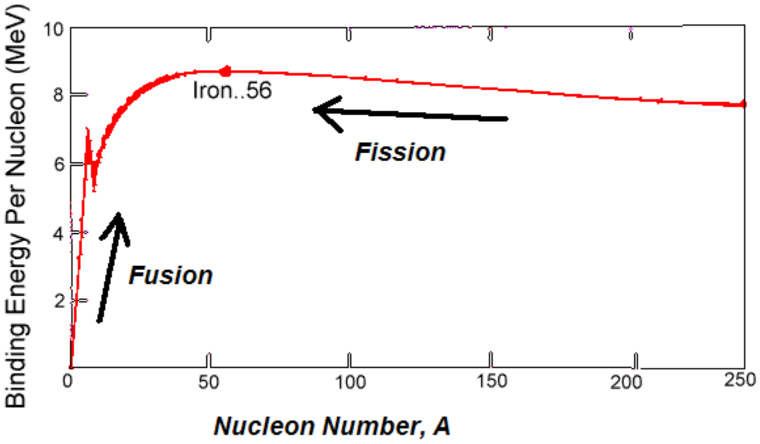
e.g. Carbon-12 nucleus has a mass of 1.9926483x10^{-26}kg and the total mass of 6 protons and 6 neutrons is 2.0085412x10^{-26}. Hence mass defect = 1.58829x10^{-28}kg.

Binding energy = Δmc^2 = 1.58829x10^{-28}kg x (3x10^8)^2 ≈1.43 x10^{-11}J

Hence the binding energy per nucleon is 1.43x10^{-11} / 12 ≈ 1.2x10^{-12}J

Hence, on average a minimum of 7.5MeV is needed by each nucleon to free itself from the strong nuclear attractive forces of the rest of the nucleon. A larger value indicates more stability.

From the graph we can see that with the greatest binding energy per nucleon, Iron 56 is the most stable nucleus.



Nuclear fission

Can occur with uranium-235.

- A *slow-moving* neutron is absorbed by a nucleus such as uranium-235.
- The resulting nucleus is very unstable and splits into two unequal nuclei and a number of *fast-moving* neutrons.

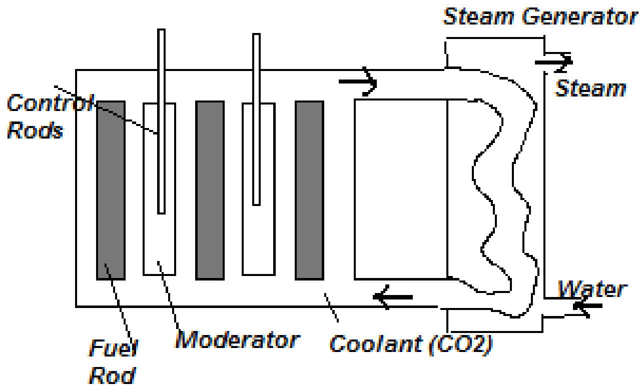
${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U^{*} \rightarrow {}^{90}_{36}Kr + {}^{143}_{56}Ba + 3{}^{1}_{0}n$ * very unstable

In this fission reaction;

- The proton number, nucleon number and mass-energy are conserved.
- The total mass of the two nuclei and neutrons (products) is LESS than the nuclei and neutron (reactants). The energy is released ($\Delta E = \Delta mc^2$) as kinetic energy of the neutrons and fragment nuclei.

In most fission reactions, there are 2/3/4 neutrons emitted. These go on to create further fission reactions causing a **chain reaction.** In a nuclear power station, the reactions are controlled (unlike a nuclear bomb).

Fission Reactors



- **Fuel Rods** contain pellets if **fissile material** (e.g. uranium, plutonium etc.)
- **Coolant** of a gas or liquid (e.g. CO_2 or H_2O). Used to remove the thermal energy produced from the fission reactions in the reactor core. The thermal energy is used to heat water and create high-pressure steam to drive the turbines of the generators.
- **Moderator** surrounds the nuclear fuel rods (graphite). Slows down the fast-moving neutrons produced in the fission reactions. The fast moving neutrons make inelastic collisions with the graphite atoms. Fast moving neutrons have a smaller chance of reacting with the uranium nuclei than the slow-moving ones.
- **Control Rods** absorb the neutrons. The control rods can be lowered to slow down the fission reactions. Made of boron (or cadmium).

Nuclear fusion

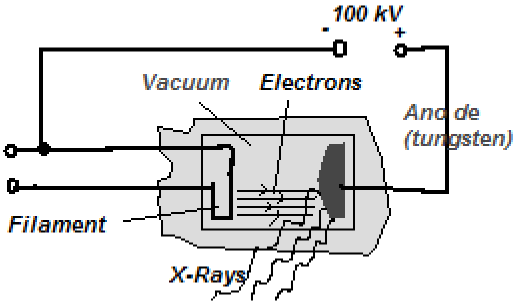
Massive nuclei such as uranium can become much more stable by either fission reactions or radioactive decay. This also holds true for small nuclei such as deuterium ${}^{2}_{1}H$

${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{4}_{2}He$

- The proton number, nucleon number and mass-energy are all conserved.
- The mass of the ${}^{4}_{2}He$ nucleus is LESS than the total mass of the two ${}^{2}_{1}H$ nuclei.
- The energy is released as kinetic energy of the ${}^{4}_{2}He$ nucleus.
- These reactions are not as easy to start as the electrically charge deuterium nuclei repel each other, so they do not easily get close enough to fuse. Temperatures of 10^7K give the nuclei sufficient kinetic energy to overcome electrostatic repulsion.

Nuclear fusion reactions occur in the cores of stars because of high temperatures and density..
Thermonuclear fusion reactions are responsible for the creation of elements such as oxygen, carbon etc.

Medical Imaging



X-Rays

X-rays are **EM waves** with a wavelength in the **range 10^{-8}m to 10^{-13}m**.

- X-rays are produced when **fast-moving electrons smash** into a 'target' metal such as **tungsten**.

  The electrons are produced at the **hot filament**. These electrons are accelerated towards the target metal, which is a high-melting point material such as **tungsten**. The **p.d.** between the **filament** (cathode) and **target metal** (anode) is about **100kV**.
  Most of the **kinetic energy** of the **electrons** is transformed into **heat** in the **metal**. The target metal gets so **hot** is **cooled down** by **circulating water**.
  About 1% is converted into **X-rays as photons**.

The **intensity** of an X-ray beam drops when passing through matter. This is equivalent to X-ray photons either being stopped or scattered by the atoms of the material. There are **three interaction mechanisms** by which X-ray photons are absorbed or scattered when passing through matter;

- **Photoelectric effect**; For photons of **<0.1 MeV Joules** of energy; Photon disappears and it's energy is used to eject an atom of the target metal
- **Pair production;** For photons of **0.5-5.0 MeV Joules** of energy; Photon loses some energy to eject an electron from the atom of the target metal. The scattered photon has less energy (greater λ).
- **Crompton effect**; For photons of **>1.02 MeV Joules** of energy. The photon disappears and produces an electron-positron pair.

The **intensity** is defined as the power per unit cross-sectional area. $I = \frac{P}{A}$

The intensity of an X-ray beam decreases exponentially with the thickness **x** of the material due to the factors above. The attenuation coefficient is μ.

$$\Longrightarrow \quad I = I_o e^{-\mu x}$$

Example
X-ray photons of energy 50keV are used when imaging the human skeleton. The attenuation coefficients for bone and muscle are 3.3cm^{-1} and 0.5cm^{-1}. Find the fraction of X-ray intensity after passing through 2.5cm of bone and muscle.
$I = I_0e^{-ux}$ $I = I_0e^{-3.3x\,2.5}$= 26%(I_0). Similarly for bone, I = 0.29%(I_0). Bones absorb more X-ray photons. Hence it is easy to view broken bones.

Detection Systems
An **intensifier** screen may be used to reduce the exposure time for an X-ray image. It consists of photographic plate between two intensifier screens. Each screen is made from a material such as Phosphor. Phosphor is a scintillator meaning the energy of a single X-ray photon on it is changed to several thousands of visible light photons, producing a brighter image ☺.
Image intensifiers may be used to produce a brighter real-time image. The expose to X-rays is reduced.

Contrast media includes materials such as iodine and **barium**. The patient swallows a liquid rich in barium and the contrast media readily **absorbs** the **X-rays** because it has a large attenuation coefficient. Used to outline the image of soft tissues, such as the **intestines**.

Computerised axial tomography (CAT)

Uses X-rays to produce 3-d images through a patient

- Patient lies on a table which moves through a **round opening** called the **'gantry'**. The ring inside the gantry contains an **X-ray tube**, which moves at **high speeds** around the **patient** and **720 X-ray detectors.**
- The X-rays are picked up by the detectors after going through the patient and the **relative intensity** is recorded. Thousands of **images** are **recorded**, taking 10 to 30 mins.

The advantages of CAT scans to single X-ray images are;

- Produce **3-d** images
- Can distinguish between tissues of **similar attenuation coefficients**.
- Show the **precise position** and **shapes** of tumours

However, single x-rays are much quicker.

Diagnostic methods in medicine

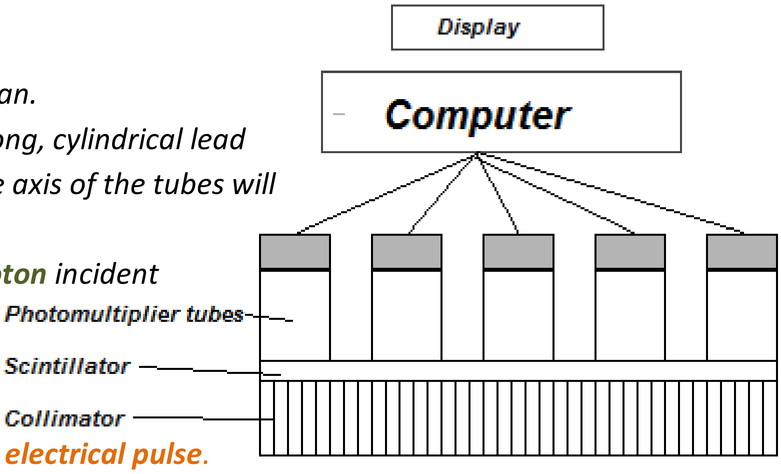
Tracers are radioactive elements that are ingested or injected.

Iodine-131 is a beta emitter with a half life of 8.1 days. Healthy kidneys will pass the iodine through to the bladder. If not the iodine builds up in the kidney. Geiger counters are used.

Technetium-99m is an extremely versatile tracer that can be used to monitor the function of the heart, liver, lungs, kidneys, brain etc. Half life is about 6 hrs, and emits gamma rays (least ionising meaning all emerge from patient). Detected with a *gamma camera*.

*A **Gamma camera** is placed above the patient's organ.*

- *The **collimator** consists of a honeycomb of long, cylindrical lead tubes. Only **gamma rays** travelling along the axis of the tubes will reach the scintillator.*
- *The **scintillator** means each **gamma ray photon** incident produces an abundance of **photons of visible light.***
- ***Photomultiplier tubes** means a single **photon of visible light** entering produces an **electrical pulse**.*
- ***Computer and display** means the **electrical pulses** from all the tubes are used to pinpoint the gamma-emitting tracer within the patient.*



Positron Emission Tomography (PET)

Uses gamma rays emerging from a tracer within a patient. Similar to CAT, producing images of slices through the patient, but used gamma rays, and not X-rays.

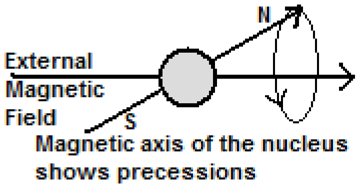
Uses fluorin-18; a positron emitter. ${}^{18}_{9}F \rightarrow {}^{18}_{8}O + {}^{0}_{+1}e + \gamma + v$

- The positrons emitted in the decay of fluorine-18 are used in PET and not the gamma ray photons at this stage. Once emitted, the positron soon interacts with an electron the two annihilate each other.

- The result of this annihilation is two gamma ray photons emitted in *opposite* directions. Detectors around the ring pick these up and build an image of the patient with the relative difference in time between the detections showing exactly where the radioactive tracer is.
- Used to show where glucose is being used in metabolic activity, giving a 3-d image

Magnetic resonance and MRI

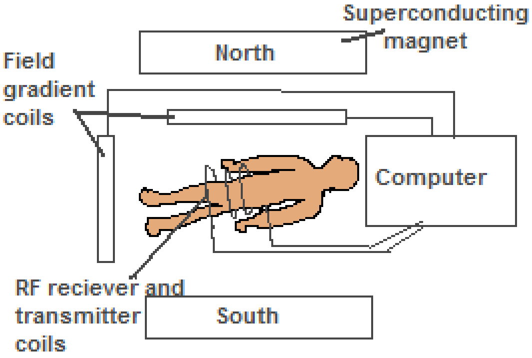
Nuclei spin about an axis. This spin makes a nucleus behave as a magnet with north and south poles. In the presence of a strong external magnetic field:



- *Most* of the nuclei align their magnetic axes parallel to the external magnetic field and a few nuclei align their magnetic axes anti-parallel to the external magnetic field (high-energy).
- The magnetic axes of the nuclei rotate about the direction of the external field. This is **precession**.
- The **angular** frequency of precession is the **Larmour frequency** $\boldsymbol{\omega_0}$**.** The precession frequency $f_0$is directly proportional to the magnetic flux density B of the external magnetic field as is given by:
- $\boldsymbol{f_0 = \frac{\gamma B}{2\pi}}$

Where γ is the gyromagnetic ratio that depends on the nucleus itself. The precession frequency for protons is about 60MHz. This lies in the range of radio waves in the EM spectrum. This is crucial.

- In a patient lying in a magnetic field, the protons precess about this field. When a radio-wave is subjected of frequency f_0, the low-energy state protons **resonate** and flip into a higher-energy state by precessing anti-parallel to the external magnetic field. When the radio frequency is turned off, they release their surplus energy as radio-wave photons of frequency f_0. The time taken for the protons to return from high to low energy state is the **relaxation time.** This depends on the tissue.



- **Superconducting magnet** provides the strong magnetic field of flux density about 2T
- **RF transmitter coil** transmits radio-frequency pulses
- **RF receiver coil** picks up the radio waves emitted by the nuclei returning to their low-energy state
- **Gradient coils** provide an additional external magnetic field along the length, depth and width of the patient. The lamour frequency is therefore slightly different for each part of the body. This identifies where the photon came from ;)
- **Computer** controls the radio-frequency pulses. Uses the different lamour frequency to pinpoint the location and relaxation time identifies the type of tissue. Produces a 3-d image.

Advantages of MRI;

- Better soft-tissue contrast than a CAT scan.
- 3-D image of the body
- Not subject to ionising radiation as with a CAT scan
- No after effects

Disadvantages of MRI

- Patients with metal objects in then cannot have scan as they get hot.
- Can take up to an hour & expensive

Ultrasound

Any sound wave with a frequency greater than the upper limit of human hearing (and hence very small wavelength).

An **ultrasound transducer** is a device for emitting and detecting ultrasound. The key component is **piezoelectric crystal** which works on the principle of the **piezoelectric effect.** The same one is used to transmit and detect.

- The crystal **vibrates** when an **alternating voltage** is applied between its ends. The **frequency** of **vibration** is the same as the frequency of the alternating voltage. Vibrations produce the **ultrasound** from the vibrating crystal of the same frequency.
- When **ultrasound** hits the same crystal, it **vibrates**. The forced **vibration** of the crystal emits an **alternating e.m.f** across its end. The **frequency** of this **induced e.m.f** is equal to that of the **induced ultrasound**.
- Hence it can act as a receiver, producing an alternating e.m.f. from ultrasound or a transmitter; producing ultrasound from an alternating e.m.f.

In an ultrasound scan, the intensity of the ultrasound reflected at a boundary between two materials is important and depends of the **acoustic impedance, Z,** of the materials.

$acoustic\ impedance = density\ of\ material\ \times speed\ of\ ultrasound\ in\ material$ i.e. $\boldsymbol{Z = \rho c}$

When reflected, the incident ultrasound at right angles to the boundary between the materials satisfies:

$\frac{I_r}{I_0} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$ where I_r is the reflected intensity, I_o the original intensity, Z_1&Z_2 the acoustic impedances of the two materials.

Example

The acoustic impedance of bone and brain tissue is $6.4 \times 10^6 kgm^{-2}s^{-1}$ and $1.6 \times 10^6 kgm^{-2}s^{-1}$. Why is an ultrasound scan not a sensible procedure for imaging a patient with head, brain injuries.

$I_r/I_0 = (6.4-1.6)^2/(6.4+1.6)^2 = 0.36$. Hence the skull will reflect a significant amount of the ultrasound intensity meaning it will not give enough details about the brain.

In a ultrasound scan, radiographers would like to distinguish between different tissue, but this is only possible.

*Z_1 and Z_2 of air and skin are **(Very low compared to air)** $400 kgm^{-2}s^{-1}$ and $1.7 \times 10^6 kgm^{-2}s^{-1}$. When the ultrasound is placed directly on skin of the patient, $I_r/I_0=0.9991$. Hence 99.91% of the ultrasound intensity is reflected back. Hence gel with a higher Z_1 value is used meaning only a small amount is reflected back.*

A-Scans are used to determine the internal dimensions of cysts, organs etc. The ultrasound transducer sends **pulses** into the patient and the **same transducer detects** the **reflections** off various boundaries.

The time taken for the ultrasound to travel **twice** the length between the material boundary (i.e. retina in the eye) is used to **measure the distance** when you know the **speed** of **ultrasound** within the **material**.

Example

The speed of ultrasound in the fluid in the eye is $1060\ ms^{-1}$. A radiographer finds that the time taken for a pulse to travel twice the length of the eyeball is $40\mu s$. Use this information to find the length of the eyeball.

$1060 \times 20 \times 10^{-6} / 2 = 2.1$ cm

B-Scans produce a detailed image of the cross-section through a patient, composed of many A-scans. The transducer is moved around the patient's body. Each reflected pulse is analysed and the depth and nature of the reflecting surfaces is determined. Generates a 2-D image of inside the patient.

The **Doppler Effect** states the wavelength (or frequency) of a wave changes when there is relative velocity between a source and detector.

- This can be used to measure **speed of blood flow** be having a transducer pointed towards a major artery. The ion-rich blood cells reflect the ultrasound. For blood travelling towards the transducer the reflected ultrasound has a slightly higher frequency. This value of Δf is directly proportional to the speed of the blood.
- The **rate of heartbeats** can be measured by placing the transducer towards the heart. The surface of the heart approaches and recedes from the transducer. The increase and decrease in the frequency of the detected ultrasound is used to determine the beat rate.

Modelling the Universe

The Structure of the Universe

- Our **solar system** consists of the Sun, planets, asteroids, planets satellites and comets.
- Our **galaxy** (on average containing 10^{12} stars) is known as the Milky Way.
- The **Universe** has about 10^{11} galaxies. It is saturated by electromagnet radiation (usually in the microwave region), interstellar dust and dark matter. Neutrinos and black holes are thought to contain the dark matter.

All the stars had the same start as a stellar.

- They begin as a large **interstellar gas cloud** consisting mainly of atoms of **hydrogen** and a few other elements (e.g. Fe)
- **Gravitational attraction** between the atoms of the dust cloud causes the cloud to **collapse**.
- The **gravitational collapse** causes the gas cloud to **heat up**.
- The atoms have greater **kinetic energy** and move faster.
- The chance of **fusion reactions** becomes greater.
- **Hydrogen nuclei** fuse together to form **helium nuclei** at temperatures of **10^7K**;
  $4\,{}^{1}_{1}H \longrightarrow {}^{4}_{2}He + 2\,{}^{0}_{+1}e + 2v$ (Known as hydrogen burning)
- Fusion reactions **increase** the **temperature** of the cloud.
- A star of stable size is formed when the gravitational pressure balances out the radiation pressure (from photons released in fusion reactions)

The size of the star depends on the mass of the initial dust cloud. The final fate of the star depends upon this mass. The core of an older star is layered with different shells of elements. When all the fuel is used up, the radiation pressure decreases. The increase in the gravitational pressure causes the helium nuclei in the outer layer to fuse together. The increase in the power production from the helium shell causes the outer layer of the star to expand due to radiation pressure.

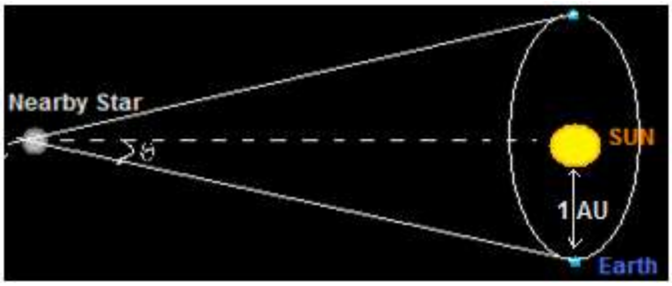
For a star of mass < 3 solar masses

- **Surface area increases** and **surface temperature drops**. It becomes a **red giant.**
- The core continues to collapse. When the temperature reaches 10^8K, the **helium** starts to **fuse**. This is known as **helium flash** and about half the material surrounding the core is ejected away as a **planetary nebula**.
- The remnant core left is a **White Dwarf** There are no further reactions inside. Glows brightly due to photons produced from past fusion reactions leaking away. High density.
- Prevented from further gravitational collapse by **electron degeneracy** or **Fermi pressure.** This comes about because two electrons cannot exist in the same quantum state. The maximum mass of a white dwarf is about 1.4 solar masses; this upper limit to the mass of a white dwarf is known as the **Chandrasekhar limit.**

For a star of mass > 3 solar masses

- **Surface area increases** and **surface temperature drops**. Becomes a **super red giant.**
- When the core collapses to form a **white dwarf**, it's mass is greater than 1.4 solar masses. The gravitational pressures are enormous and overcome the **Fermi pressure**. The **electrons** within the core **combine** with **protons** to produce **neutrons** and **neutrinos**. The **neutrinos escape** and the central core becomes entirely packed within **neutrons**.
- The **outer shells** surrounding the neutron core rapidly **collapses** and rebound against the solid **neutron core**. This generates a **shock wave**, which explodes the **surface layers** of the star as a **supernova**.
- The supernova **blasts off heavier** elements like iron and oxygen into space. (All elements originate from supernovae)
- For stars with mass in the range 3-10 solar masses, the remnant core is a **neutron star.** For stars of above 3 masses, the neutron core continues to collapse into a **black hole.**

An **astronomical unit** is the mean distance between the Earth and the Sun. 1 AU ≈ 1.5 x 10^{11}m.
The **light-year** is the distance travelled by light in a vacuum in a period of 1 year. 1 ly ≈ 9.5 x 10^{15}m.



The **Stellar parallax**: Can be used to calculate the distance a star is away. The *parallax* is the angle θ.
Arc seconds is a unit of measurement of an angle.
1 arc second = $\frac{1}{3600}$degrees

A parsec is the distance that gives a parallax angle of 1 second of arc (or 1/3600 degrees).
1 pc ≈ 3.1 x 10^{16}m

A relationship between distance *d* in pc and parallax *p* in arc seconds is: $d = \frac{1}{p}$

Example
Sirius is a very bright star in the night sky. It has a parallax of 0.38 arc sec. Find the distance in parsecs and light years.
d = 1/(0.38) = 2.63 pc. 1pc = 3.1x10^{16}*m and 1ly= 9.5x*10^{15}*m => 2.63 x (3.1x*10^{16}*)/(9.5x*10^{15}*) = 8.6 ly*

Olbers' paradox states that for an **infinite**, **uniform** and **static** universe, the night sky should be bright because of light received from stars in all directions.

- In an infinite universe, the number of stars in a spherical shell increases with distance^2.
- The light received from each star decreases with distance^2.

Olber's paradox is based on incorrect assumptions. The universe is **neither static nor infinite**. The fabric of space, which includes the galaxies, has been expanding since the big bang. Hence:

- **The universe is finite in size**
- The universe is **not static** but **expanding** (as confirmed by redshift of light from distant galaxies and Hubble's law)
- The **finite age** of the **universe** means that light from distant galaxies has not reached us yet.

The **Doppler effect** can also be observed in the starlight from distant galaxies. Light emitted from a star moving away from us will have a longer wavelength. The entire spectrum from a star is shifted by the same fraction to longer wavelengths which is known as **redshift**. If a star is moving towards us, then the entire spectrum is shifted to shorter wavelengths which is known as **blueshift**.

The speed *v* of a star can be determined by measuring the wavelength of known spectrum lines using the Doppler equation; $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

Example
The wavelength of a particular spectral line in the laboratory is 119.5 nm. The same spectral line emitted from a star has a wavelength of 121.6 nm. Find the speed of the star and state whether it is moving away or coming towards us.
$(121.6-119.5)\times10^{-9}/(119.5\times10^{-9})\times3\times10^{8} = 5.3\times10^{6}\ ms^{-1}$ *hence it is moving away from us.*

All galaxies are moving away from each other because of space expanding. **Hubble's law** states the **recessional speed of a galaxy is directly proportional to its distance from us.**

$\Longrightarrow v \propto x$

$\Longrightarrow \boldsymbol{v = H_0 x}$ where H_0 is the Hubble Constant.

$$Age\ of\ the\ Universe = \frac{1}{H_0}$$

Example
A galaxy at a distance of 100 Mpc has a recessional speed of 8000 kms^{-1}. Use this information to determine the Hubble constant in s^{-1} and estimate the age of the universe in years.
$v = H_0 x\ 8000kms^{-1} = H_0(100\times10^{6}\times3.1\times10^{16}) \Rightarrow H_0 = 2.58\times10^{-18}s^{-1}$. *Age* $= 1/H_0 = 3.88\times10^{17}$ *s = 12 billion Yrs.*

The Evolution and Fate of the Universe

The **cosmological principle** states the universe is **homogeneous** (on a large scale, density is evenly distributed), **isotropic** (the same in all directions) and has **universal laws of physics** (laws of physics can be applied everywhere).
The **big bang** model assumes space and time evolved from a *singularity* in an even that took place 12 billion years ago.

- It was infinitesimally **small**, infinitely **dense** and very **hot**. All **four forces** (**gravitational**, **electromagnetic**, **strong nuclear** and **weak nuclear**) were **united**.
- An expansion of the universe led to its cooling
- At **10^{-6}s**, the temperature of the universe was about **10^{14}**K. The universe consisted of energetic **quarks** and **leptons**.
- At **10^{-3}s** the temperature of the universe was about **10^{12}** K. The **strong nuclear force** became dominant and **combined** the **quarks** to form **hadrons** (including protons and neutrons).
- At **10^{7}** K **fusion** reactions between **protons** produce **helium nuclei**.
- At **10^{4}** K, electrons combined with nuclei to form **hydrogen** and **helium** atoms.
- **Gravitational forces** become dominant. **Hydrogen** and **helium** clump together to form **stars** and eventually clusters of galaxies.
- The temperature of the universe is now **2.7K**. It is saturated with EM waves of the **microwave** region. On the Earth, this background microwave radiation is isotopic.

Evidence for the Big Bang

- The universe is expanding
- *Hubbles's law* shows galaxies are receeding from us.
- The temperature of the universe is 2.7K (with small ripples).
- The universe is **saturated** with **background microwave radiation**
- The most distant galaxies (and hence the youngest) show a chemical composition of 25% helium.

The final fate of the universe depends on its density. This is hard to calculate because it includes dark matter. The fate of the universe depends upon the **critical density** ρ_0;

$\rho_0 = \frac{3{H_0}^2}{8\pi G} \approx 1.2 \times 10^{-26}\ \mathrm{kgm^{-3}}$ (as we don't know the exact value of H_0)

- [**Closed** Theory] If the density is **higher** than ρ_0 then the gravitational force between matter will be strong enough to decelerate, halt and contract the expansion of the universe, getting hotter as it approaches the **big crunch**. The universe will then oscillate between big bangs and big crunches.
- [**Flat** Theory] If the density is **equal to** ρ_0 then the rate of expansion of such a universe will tend to zero and the volume tend to a limit. (Most cosmologists believe this to be the case)
- [**Open** Theory] If the density is **less** than ρ_0 the gravitational force between matter in such a universe is too weak to decelerate the expansion of the universe. The universe will expand forever as the temperature decreases resulting in the **big freeze**.

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